A low-friction high-load thrust bearing and the human hip joint. Part 2.

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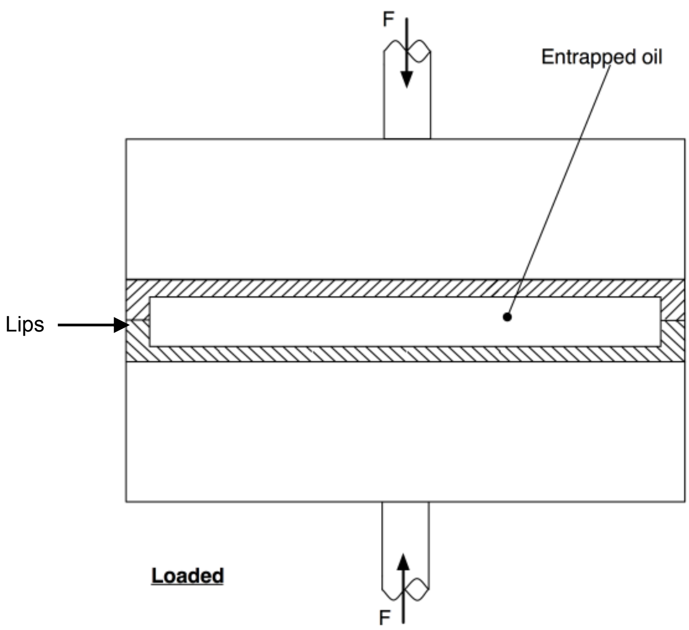
ABSTRACT

The paper, McIlraith 2010, "**A low-friction high-load thrust bearing and the human hip joint**" (referred to below as **PART 1**), described a hydrostatic thrust bearing operating at a pressure of 130 MPa (1300 bar) and with a coefficient of friction rising to 0.004 in 6 days. It consisted of interleaved oil-coated Mylar and brass shims, each 0.1 mm thick. At this pressure the Mylar (Young's modulus 3000 MNm^{-2})* deforms to reveal a pool of lubricant bounded by contacting layers of shims at its edges where the pressure tapers off to zero. Thus, most of the load is borne by the oil so its effective Coulomb (slip-stick) friction is very low. It is suggested that the human hip joint, a bearing with similar geometry and lined with soft material, articular cartilage, operates at 1 bar in a manner similar to the plastic bearing operating at 1300 bar.
*A more accurate value is 4000 MN m^{-2} and is used below.
The original experiment was performed with a flat thrust bearing.
Attention has been drawn to the possibility that in the postulated model of a thrust bearing lined with soft material the applied pressure might burst the structure open.
An investigation of the conditions governing this 'bursting' effect has led to a deeper understanding and strengthening of the original model.
This new knowledge has been applied successfully to the original experiment: to a suspected case of bursting: and to revealing the conditions needed for operation at atmospheric pressure.

In addition, this work makes possible the rational design of a wide range of engineering thrust bearings with extremely low friction operating at pressures down to 1 bar.

INTRODUCTION

My attention has been drawn by Professor Jeffery L. Tallon of Victoria University, N.Z. to an important effect illustrated in **Figure 1** (right) which is a simplified version of figure 2 in PART 1. It is that, as drawn, the pressure of the entrapped oil might be high enough to burst open the ends, a problem I had not considered.



It follows that the present task is to find a condition such that the pressure is high enough to make the compression $\nabla \ell$ of the soft material greater than its surface roughness R (for low friction operation), and yet low enough to avoid bursting.

The aim of this work is not to get exact solutions, a difficult task, but to find approximate solutions which reveal the main dominating factors and the relationships between them.

This paper is divided into two parts. The first part deals with the conditions needed for low friction operation. The second part deals with bursting.

CONDITIONS FOR LOW FRICTION IN FLAT THRUST BEARINGS

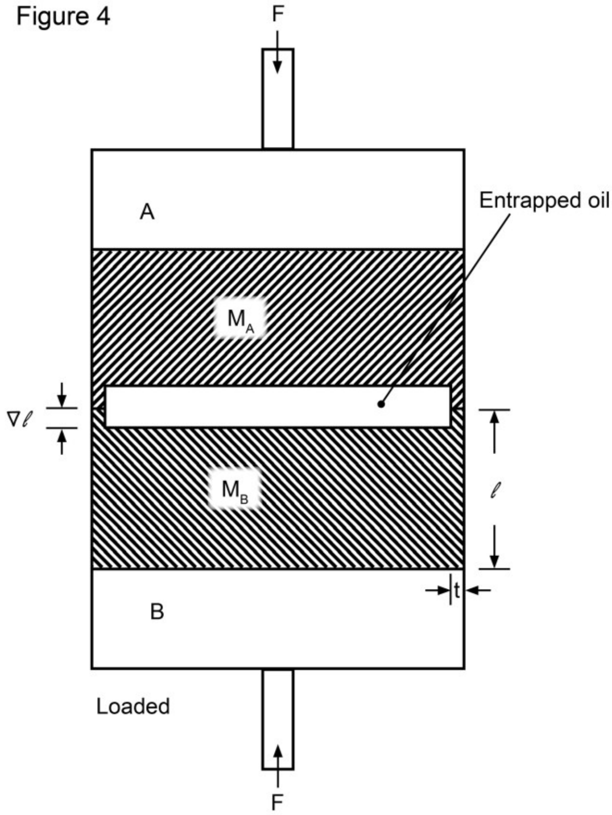
We know from the original experiments on a flat thrust bearing comprised of interleaved shims of brass and Mylar immersed in high viscosity oil, PART 1, that the Coulomb (slip-stick) friction opposing rotation of the bearing was very low. Even after 6 days with the original pressure P_0 at 130 MPa (1300 bar), it had built up to only 0.004 with no sign of bursting.

Therefore, we must have had an extensive pool of oil deeper than the roughness of the opposing surfaces to allow low-friction operation.

Also this pool must have been bounded by a dam strong enough to withstand the above pressure, yet thin enough to offer minimal opposition to rotation between opposing rubbing surfaces. There was no bursting so the outwards movement q of the dam due to the oil pressing against it must have been less than its thickness t.

This situation is well portrayed by Figure 4. (Figure 1 is in Part 2, Figures 2 and 3 are in Part 1 while Figure 4 is in Part 2.)

Figure 4



The present task is to find under what circumstances similar behaviour can be exhibited at various pressures down to atmospheric pressure.

The first part of the model being developed is shown in Figure 4. It is similar to Figure 2 (Part 1) but with much less compression of the soft material.
It consists of two solid circular cylinders, A and B, of radius r. Each is lined with soft elastic material, M_A and M_B, of thickness ℓ.

Between A and B, oil at pressure P_F causes compression $\nabla\ell$ of the soft material, i.e $\nabla\ell = \ell P_F / Y$. $P_F = F / \pi r^2$ where F is the applied axial force. Y is Young's modulus of the soft material.

At the edges of A and B the pressure falls to zero so vertical lips of length $\nabla\ell$ of soft material are formed.

Does this model fit the original experiment?

In the original experiment very low friction and no bursting were experienced.

Operating conditions were:
P_F (=P_0)=130 MPa ; Y=4000 MNm^{-2}; R = 2.5 x 10^{-4} mm (measured); ℓ = 0.1 mm. At this pressure $\nabla\ell$ (=$\frac{P_F \, . \, \ell}{Y}$) = $\frac{130 \times 0.1}{4000}$ mm This is greater than the measured value of the surface roughness R by the factor $\frac{P_F \times \ell}{Y \times R} = \frac{130}{4000} \times 0.1 \times \frac{1}{2.5 \times 10^{-4}} = 13$

Thus at P_0 = 130 MPa, $\nabla \ell$ is 13 times larger than R so there is very low friction.

In the case of the human hip the soft material is articular cartilage which has Y lying in the range 0.5 to 1.0 MNm^{-2}. Therefore, with R=2.5 x 10^{-4} mm, with ℓ =1mm, with P_F = 0.1 MPa, and with Y = 1.0 MNm^{-2},
$P_F \times \ell / RY = 0.1\times10^6 \times 1 /(2.5 \times 10^{-4} \times 10^6) = 0.1 \times10^4 /2.5 = 400$

Thus, assuming the hip joint is a flat bearing, we find $\nabla \ell$ is 400 times larger than R so there is very low friction there.

FLAT BEARING AT 1 BAR

For an engineer aiming to make a flat bearing using soft material for low friction and operating near atmospheric pressure we reduce P_F to 0.1 MPa (1 bar). For low friction, the compression $\nabla \ell$ must be greater than the surface roughness R,
i.e. $\frac{P_F \ell}{Y} > R$ or $\frac{P_F \ell}{Y R} > 1$

Available soft materials will have similar values of R and Y to those of Mylar, so it is simpler to change the value of ℓ to achieve the above inequality. Let ℓ=20 mm. Then, for the engineer the ratio $\frac{P_F \times \ell}{YR}$ becomes 2.

This value of ℓ is probably OK from an engineering point of view.

Now we deal with the question of bursting.

CONDITIONS FOR FORMATION OF A STABLE POOL OF OIL WITH NO BURSTING

The oil presses horizontally against these lips causing their ends to move outwards a distance q. For stability or no bursting, q must be less than the thickness t of the dam.

Treatment of Lips as Cantilevers

We note that each lip resembles a cantilever, a well-known type of beam in mechanical engineering. A particular form of cantilever is a beam of length L which is held rigidly at one end and bends under its own weight. This allows us to adapt a standard expression for the deflection of the end of such a cantilever, with distributed weight replaced by distributed pressure. In this case, to a first approximation, we treat a lip as a cantilever of length L held rigidly at the surface of the entrapped oil (so that L= $\nabla \ell$) and bending under distributed pressure. By this means we transform a difficult problem into one which is accurate enough for present purposes.

A true cantilever is solid and is held rigidly, so that there is no movement at its base when it is loaded. The cantilever postulated here is made of soft material and is held by the same soft material with the result that its effective base is a small distance below the surface of the oil.

More importantly, with changing pressure, a small rotational movement θ about its base occurs. This is taken into account below by modifying the standard expression for a cantilever bending under its own weight.

In the case of a cantilever of constant cross section bending under its own weight, the standard expression for it is given by
$q = (m\,L^4)/(8YI)$

where
q is the deflection at its end
m = weight per unit length of the cantilever
L is the length of the cantilever
Y is Young's modulus of the cantilever
I is the moment of inertia of the cross section of the cantilever
$I = bt^3/12$
where b is the width of the cantilever and t is its depth.

Now replace distributed weight by distributed pressure.
The total force acting on
the cantilever due to pressure P_F is $P_F\,bL$

$$\therefore\ m = \frac{P_F\,bL}{L} = P_F\,b$$

$$\therefore\ q = \frac{P_F\,bL^4}{8YI} = \frac{P_F\,bL^4 12}{8Ybt^3} = \frac{12\,P_F\,L^4}{8Yt^3} = \frac{1.5\,P_F\,L^4}{Y\,t^3}$$

The relative displacement of the cantilever at its end is given by

$$q/t = \frac{1.5\,P_F\,(L^4/t^4)}{Y}$$

But, as noted above, to a first approximation $L = \nabla\ell$

$$\therefore\ q/t = \frac{1.5\,P_F}{Y}\,\frac{\nabla\ell^4}{t^4}$$

This is a general expression for the movement at the end of a cantilever acted on by a distributed pressure.

ORIGINAL EXPERIMENT (Part 1)

According to this theory, which treats the lips as cantilevers, we get no bursting provided P_F is less than the bursting pressure P_B..

$$P_B = \frac{1}{1.5} \times Y \times (t^4/\nabla\ell^4).$$

The values used for P_B are those recorded in the original experiment.
w or t (as used here) = 0.28mm and
$\nabla\ell = h = 3.3 \times 10^{-3}$mm (corrected for larger value of Young’s modulus – see above).

This analysis is correct for that part of the soft cantilever
entirely surrounded by fluid. To take into account the rotational effect noted above, we use the modified expression $q/t = \frac{1.5\ P_F}{Y_s} (L^4/t^4 + L\theta/t)$

These components behave quite differently. The first leads to
$P_B/P_0 = 1.06 \times 10^9$ where P_0 is the operating pressure of the original experiment. The second leads to $P_B/P_0 = 20.52$.
It acts by rotating the soft cantilever as a whole through an angle θ so that the added movement at its free end is $L\theta$. So
$q/t = L\theta / t = (1.5P_F / Y_s) \times L\theta/t$. Bursting occurs when $L\theta = t$. Then
$1 = (1.5\ P_B\ / Y_s) \times 1$ or

$$P_B = Y_s/1.5 = 4000/1.5 = 2667 \text{ M Pa}$$

$$\therefore\ P_B / P_0 = 2667 / 130 = 20.52$$

It is assumed above that the soft cantilever is held by a block of material of modulus Y_s. In fact it is held less strongly, with the result that θ is a little larger and P_B/P_o is slightly smaller.

FIGURE 1

See Figure 1, above.
$P_B = 2667$ MPa (as for the original experiment).

Figure 1, as drawn, has $\frac{t}{\nabla \ell} = 1.0$ and $\frac{\nabla \ell}{\ell} = \frac{3}{4}$

The pressure P_3 needed to produce Figure 1 is given by
$P_3 = \frac{\nabla \ell}{\ell} Y = \frac{3}{4} \times 4000 = 3000$ MPa
This exceeds P_B.

Therefore bursting is bound to occur, and figure 1, as drawn, represents a non-physical situation.

An aside
The model proposed in Part1 assumes that there is lateral leakage of fluid through the articular cartilage. Experimental evidence (Lewis and McCutchen,1959, Gwynn et al, 2000) shows that the cartilage is honeycombed with non-communicating pores which are oriented perpendicularly to the bone. Therefore, contrary to the statement made in Part 1, section 4.1, lateral leakage through the cartilage does not occur. Thus, in

effect, the action of the cartilage at the boundary is closely similar to that of the Mylar in the original experiment, so the above treatment is justified.

CONCLUSION

The treatment of the lips as cantilevers has enabled the conditions governing bursting to be expressed in terms of their dimensions and their Young's moduli. This, along with the expression for low friction operation and the treatment of the hip joint as a flat bearing, has put on a firm footing the suggestions by McIlraith (2010) for a radically new model of human and animal synovial joints.

In addition, this work makes possible the rational design of a wide range of engineering thrust bearings with extremely low friction operating at pressures down to 1 bar.

On the basis of the findings recorded here we can say that the necessary conditions for a very low friction flat thrust bearing are:

1. That the two halves of the bearing be coated with a soft material which deforms under the applied pressure by more than the surface roughness.
2. At the boundary of the bearing where the pressure falls to zero, contacting layers of soft material form a dam strong enough to withstand this pressure yet thin enough to offer minimal opposition to rotation between opposing rubbing surfaces.

If the pressure is too low the deformation of the soft material is less than the surface roughness. If it is too high the dam is ruptured, i.e. bursting occurs.

References

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